Code Name:

Fall 2009 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may NOT use your book, notes, calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

Problem 1. Carefully define the following terms:

a. proposition

b. tautology

c. counterexample

Problem 2. Carefully define the following terms:

a. converse

b. contrapositive

c. vacuous proof

Problem 3. Carefully define the following terms:

a. subset

b. power set

c. symmetric difference

Problem 4. Carefully define the following terms:

a. binary relation

b. equivalence relation

c. partial order

Problem 5. Carefully define the following terms:

a. floor

b. injection

c. least upper bound

Problem 6. Carefully define the following terms:

a. event

b. graph

c. tree

Problem 7. Construct the truth table for the proposition $(p \to r) \leftrightarrow (q \to r)$.

Problem 8. We roll two identical ordinary dice, and add the results. What is the probability of getting 4?

Problem 9. Find an Eulerian path in $K_{2,3}$.

Problem 10. Find all posets on $A = \{x, y\}$.

Problem 11. Let A, B be two sets. Prove that if $A \subseteq B$ then $B^c \subseteq A^c$.

Problem 12. Prove that $n^2 \in O(n^3 - 100)$.

Problem 13. Prove that $\log_2(n) \in \Theta(\log_3(n))$.

Problem 14. Prove that $2^n = \sum_{k=0}^{n} C(n, k)$.

Problem 15. All bizzles are azzles, but not all azzles are bizzles. There is at least one cozzle that is also a bizzle. There is at least one dizzle that is not a bizzle. All cozzles are dizzles. Must there be a dizzle that is also an azzle?

Problem 16. We define a lattice on $A = \mathbb{R} \times \mathbb{R}$ as follows. For $x = (x_1, x_2), y = (y_1, y_2)$, elements of A, we say $x \leq y$ if $\big(x_1 \leq y_1$ AND $x_2 \leq y_2\big)$. For $x = (0.5, 3), y = (4, 1)$, find l.u.b.(x, y) and g.l.b.(x, y).

Problem 17. Prove that $x^2 + 2x < 8$ if and only if $|x + 1| < 3$.

Problem 18. Let $S = \{1, 2, 3, \ldots, 12\}$, an equiprobable sample space. Let A be the event of selecting a number less than or equal to 6, and let B be the event of selecting a number that is a multiple of 3. Determine whether A, B are independent.

Problem 19. Find all trees with vertex set $V = \{v_1, v_2, v_3\}$.

Problem 20. Find all simple graphs with vertex set $V = \{v_1, v_2, v_3\}$.